

FIRST TERM EXAMINATION

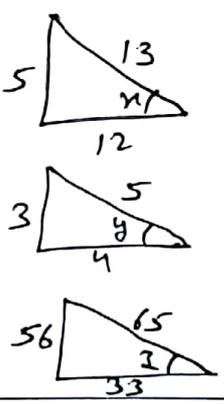
2018-2019

CLASS XII

Marking Scheme – MATHEMATICS [THEORY]

Q.NO.	Answers	Marks (with split up)
1	$(-5) * (2 * 0) = (-5) * (2) = -5 + 4 * 4 = 11$	1
2	$\tan^{-1}(-1) = -\frac{\pi}{4} \in (-\frac{\pi}{2}, \frac{\pi}{2})$	1
3	$\tan^{-1}x + \cot^{-1}\frac{3}{4} = \sin^{-1}1 = \frac{\pi}{2}$ $\therefore x = \frac{3}{4}$	1
4	$\lim_{x \rightarrow 0} (\frac{\sin 3x}{3x} + \cos x) = f(0) \therefore f \text{ is cont.}$ $\frac{5}{3} + 1 = k \Rightarrow k = \frac{8}{3}$	1
5	<u>For comm.</u> $a * b = a + b + ab$ for $a, b, c \in \mathbb{R} - \{-1\}$ $b * a = b + a + ba \Rightarrow a * b = b * a \Rightarrow *$ is comm. <u>For assoc.</u> $(a * b) * c = a * (b * c) = a + b + c + abc$ $(a * b) * c = (a + b + ab) * c = a + b + c + abc$ $\therefore a * (b * c) = (a * b) * c \Rightarrow *$ is associative	1 1
6	$f(x) = \frac{x}{x+1}$. let $x_1, x_2 \in [0, \infty]$ and $f(x_1) = f(x_2)$ $\Rightarrow \frac{x_1}{x_1+1} = \frac{x_2}{x_2+1} \Rightarrow x_1 = x_2$ $\therefore f$ is 1-1	$\frac{1}{2}$ 1 $\frac{1}{2}$
7	$\tan \{2 \tan^{-1}(0.2) - \frac{\pi}{4}\} = \tan \{2 \tan^{-1}\frac{1}{5} - \tan^{-1}1\}$ $= \tan [\tan^{-1}\frac{5}{12} - \tan^{-1}1] = \tan \tan^{-1}(\frac{\frac{5}{12} - 1}{1 - \frac{5}{12}})$ $= -1$	$\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$

8	$LHD = \lim_{x \rightarrow 2^-} \frac{ x-2 - 2-2 }{x-2} = \lim_{x \rightarrow 2^-} \frac{-(x-2)}{x-2} = -1$ $RHD = \lim_{x \rightarrow 2^+} \frac{ x-2 - 2-2 }{x-2} = \lim_{x \rightarrow 2^+} \frac{(x-2)}{x-2} = 1$ <p>$\therefore LHD \neq RHD \therefore f$ is not derivable at $x=2$</p>	1 $\frac{1}{2}$ $\frac{1}{2}$
9	<p>let $y = \sin(\cos^2 \sqrt{x})$</p> $\frac{dy}{dx} = \cos(\cos^2 \sqrt{x}) \cdot (2 \cos \sqrt{x}) \cdot (-\sin \sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$ $= \frac{-\cos(\cos^2 \sqrt{x}) \cdot \cos \sqrt{x} \sin \sqrt{x}}{\sqrt{x}}$	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$
10	$\frac{dy}{dx} = (x-1)^{2/3} \cdot \frac{1}{3}(x-1)^{-2/3} + (x+1)^{1/3} \cdot \left(\frac{2}{3}(x-1)^{-1/3}\right)$ $= \frac{3x+1}{3(x^2-1)^{1/3}(x+1)^{1/3}}$ <p>At $x=0$</p> $\frac{dy}{dx} = \frac{1}{3(-1)(1)} = -\frac{1}{3}$	1 $\frac{1}{2}$ $\frac{1}{2}$
11	$y = f\left(\frac{2x+3}{3-2x}\right) = \log\left(\frac{2x+3}{3-2x}\right) = \log(2x+3) - \log(3-2x)$ $\frac{dy}{dx} = \frac{1 \times 2}{2x+3} - \frac{1 \times (-2)}{3-2x}$ $= \frac{12}{9-4x^2}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
12	<p>$V_{\text{cube}} = x^3$ and $A_{\text{cube}} = 6a^2$;</p> $\frac{dV}{dt} = 3x^2 \frac{dx}{dt} \Rightarrow 9 = 3x^2 \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{3}{100} \text{ cm}^3/\text{s}$ <p>now</p> $\frac{dA}{dt} = 12x \frac{dx}{dt} = 12 \times 10 \times \frac{3}{100} = 3.6 \text{ cm}^2/\text{s}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
13	<p><u>For transitive</u> $(1,2) \in R ; (2,3) \in R \Rightarrow (1,3) \in R$ ①</p> <p><u>For symmetric</u> $(1,2) \in R \Rightarrow (2,1) \in R$ and $(2,3) \in R \Rightarrow (3,2) \in R$ ②</p> <p>From ① $(1,3) \in R \Rightarrow (3,1) \in R$</p>	$\frac{1}{2}$ 1 $\frac{1}{2}$

	<p>Also by ② $(1,2) \in R \Rightarrow (2,1) \in R \quad \therefore$ for transitive $(2,2) \in R$</p> <p>hence list of additional element of R is $(2,1), (3,2), (3,1), (2,2)$</p> <p>$R = \{ (1,2), (1,1), (2,3), (2,1), (3,2), (3,1), (2,2) \}$</p>	<p>1</p> <p>1</p>
14	<p>a) Clearly f is 1-1 as well as onto but g is neither 1-1 nor onto $\therefore 3 \in T$ does not have a pre image in S also $a, c \in S; (a,1), (c,1) \in R \quad \therefore g$ is not onto</p> <p>b) $f^{-1} = \{ (3,a), (2,b), (1,c) \}$ g^{-1} does not exist as it is neither 1-1 nor onto</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>
15	<p>$y = \cos^{-1} x + \cos^{-1} \left\{ \frac{1}{2} \cdot x + \frac{\sqrt{3}}{2} \sqrt{1-x^2} \right\}$; put $x = \cos \theta; \frac{1}{2} = \cos \alpha \Rightarrow \alpha = \frac{\pi}{3} \quad \therefore \frac{\sqrt{3}}{2} = \sin \alpha$</p> <p>$y = \cos^{-1} x + \cos^{-1} \{ \cos \alpha \cos \theta + \sin \alpha \sin \theta \}$ $= \cos^{-1} x + \cos^{-1} [\cos (\alpha - \theta)] = \cos^{-1} x + \alpha - \theta$ $= \cos^{-1} x + \cos^{-1} \frac{1}{2} - \cos^{-1} x$ $y = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>
16	<p>LHS = $\cos^{-1} \left(\frac{12}{13} \right) + \sin^{-1} \left(\frac{3}{5} \right)$ $= \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{3}{4}$ $= \tan^{-1} \frac{56}{33}$ $= \sin^{-1} \frac{56}{65} = \text{RHS}$</p> <p>Hence proved</p>	 <p>1+1</p> <p>1</p> <p>1</p>
17	<p>$\cos \left[\cos^{-1} \left(\frac{\sqrt{3}}{2} \right) + \frac{\pi}{6} \right] + \sin \left[\frac{\pi}{2} - \sin^{-1} \frac{\sqrt{3}}{2} \right]$</p>	

	$\cos\left[\pi - \frac{\pi}{8} + \frac{\pi}{8}\right] + \sin\left[\frac{\pi}{2} - \frac{\pi}{3}\right]$ $-1 + \cos\frac{\pi}{3}$ $-1 + \frac{1}{2} = -\frac{1}{2}$	2 1 1
18	<p>At $x=1$</p> $\text{LHD} = \lim_{x \rightarrow 1^-} \frac{x-1}{x-1} = 1$ $\text{RHD} = \lim_{x \rightarrow 1^+} \frac{(2-x)-1}{x-1} = \frac{1-x}{x-1} = -1$ <p>f is not derivable at $x=1$</p> <p>At $x=2$</p> $\text{LHD} = -1$ $\text{RHD} = -1$ <p>$\therefore \text{LHD} = \text{RHD} \Rightarrow f$ is diff. at $x=2$</p>	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
19	$f'(x) = 2\cos x + 2\cos 2x$ <p>Clearly f is derivable in $(0, \pi)$ and f exist at $x=0$ and $x=\pi$</p> <p>differentiability \Rightarrow continuity</p> <p>Hence f is cont. on $[0, \pi]$</p> <p>now $f'(c) = \frac{f(\pi) - f(0)}{\pi - 0} \Rightarrow 2\cos c = -2\cos 2c \Rightarrow c = \frac{\pi}{3}$ $c \in (0, \pi)$</p> <p>OR \therefore LMVT is verified.</p>	1 1 1 1
	$f(x) = 3 + (x-2)^{2/3}$ is cont. $\forall x \in [1, 3]$ $f'(x) = 3 + \frac{2}{3}(x-2)^{-1/3} = 3 + \frac{2}{3(x-2)^{1/3}}$ Clearly $f'(x)$ does not exist at $x=2 \in (1, 3)$ \therefore RT is not applicable	1 2 1
20	$\frac{dy}{dx} = \frac{1}{1 + \left(\frac{a}{x}\right)^2} \left(-\frac{a}{x^2}\right) + \frac{1}{2} \left[\frac{1}{x-a} - \frac{1}{x+a} \right]$ $= \left(\frac{x^2}{a^2 + x^2}\right) \left(-\frac{a}{x^2}\right) + \frac{1}{2} \left[\frac{x+a - x+a}{x^2 - a^2} \right]$ $= \frac{-ax^2 + a^3 + a^3 + ax^2}{x^4 - a^4} = \frac{2a^3}{x^4 - a^4}$	2 1 1
	<p>Taking log on both sides $\log(x-y) \cdot e^{\frac{x}{x-y}} = \log a$</p> $\log(x-y) + \frac{x}{x-y} = \log a$ $\frac{1-y}{x-y} + \frac{(x-y) - x(1-y)}{(x-y)^2} = 0$	1 1
22	$y \frac{dy}{dx} + x = 2y$	2

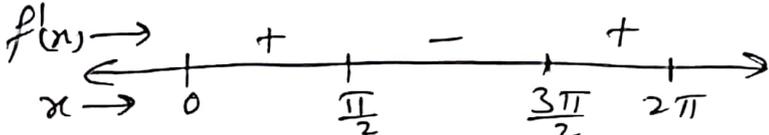
	$y = \sin^{-1} \left[(2x) \left(\frac{3}{5} \right) - \frac{4}{5} \sqrt{1-(2x)^2} \right]$	$\frac{1}{2}$
21	<p>let $2x = \sin \theta \Rightarrow \sqrt{1-(2x)^2} = \cos \theta$ and $\frac{3}{5} = \cos \alpha \Rightarrow \frac{4}{5} = \sin \alpha$</p> <p>$\therefore y = \sin^{-1} \sin(\theta - \alpha) = \sin^{-1} 2x - \alpha$</p> $\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}} - 0 = \frac{2}{\sqrt{1-4x^2}}$	<p>1</p> <p>1</p> <p>$1 + \frac{1}{2}$</p>
22	<p>$f'(x) = -1 + 2 \cos x$ now $f'(x) = 0 \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$</p> <p>$f''(x) = -2 \sin x$</p> <p>At $x = \frac{\pi}{3}$; $f''(x) = -2 \sin \frac{\pi}{3} = -\sqrt{3} \therefore x = \frac{\pi}{3}$ is point of local max.</p> <p>At $x = \frac{5\pi}{3}$; $f''(x) = -2 \sin \frac{5\pi}{3} = \sqrt{3} \therefore x = \frac{5\pi}{3}$ is point of local min.</p> <p>now $f(\frac{\pi}{3}) = \sqrt{3} - \frac{\pi}{3}$ and $f(\frac{5\pi}{3}) = -(\frac{5\pi}{3} + \sqrt{3})$</p> <p>max. value = $\sqrt{3} - \frac{\pi}{3}$ and min. value = $-(\frac{5\pi}{3} + \sqrt{3})$</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
23	<p>Given $\frac{dx}{dt} = -5 \text{ cm/s}$ and $\frac{dy}{dt} = 4 \text{ cm/s}$</p> <p>now $p = 2(x+y) \Rightarrow \frac{dp}{dt} = 2 \left(\frac{dx}{dt} + \frac{dy}{dt} \right) = 2(-5+4) = -2 \text{ cm/s}$</p> <p>$A = xy \Rightarrow \frac{dA}{dt} = x \frac{dy}{dt} + y \frac{dx}{dt} = 8 \times 4 + 6 \times (-5)$</p> $= 32 - 30$ $= 2 \text{ cm}^2/\text{s}$	<p>1</p> <p>$\frac{1}{2} + 1$</p> <p>$\frac{1}{2} + 1$</p>
24	<p>For reflexive</p> <p>For symmetric</p> <p>For transitivity</p> <p>concluding statement</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>
25	<p>$f \circ g(x) = x - x + x - x = \begin{cases} 0, & x \geq 0 \\ -4x, & x < 0 \end{cases}$</p> <p>$\therefore f \circ g(-3) = -4(-3) = 12$</p> <p>$f \circ g(5) = 0$</p> <p>$g \circ f(x) = x + x - [x + x] = \begin{cases} 2x - 2x & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$</p> <p>$g \circ f(-2) = 0$</p>	<p>2</p> <p>1</p> <p>2</p> <p>1</p>

OR

	<p>Proving 1-1 Showing $x = \frac{3-4y}{3y-4}$; where $y = \frac{4x+3}{3x+4}$; $y \in \mathbb{R} - \{\frac{4}{3}\}$ Showing onto and declaring bijective $f^{-1}(x) = \frac{3-4x}{3x-4} \Rightarrow f^{-1}(0) = \frac{3}{-4}$ $f^{-1}(2) = 2 \Rightarrow \frac{3-4x}{3x-4} = 2 \Rightarrow x = \frac{11}{10}$</p>	<p>1 $1 + \frac{1}{2}$ $1 + \frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$</p>
26	<p>$\frac{dx}{d\theta} = -a \sin\theta + b \cos\theta$ $\frac{dy}{d\theta} = a \cos\theta + b \sin\theta$ $\frac{dy}{dx} = \frac{a \cos\theta + b \sin\theta}{b \cos\theta - a \sin\theta} = \frac{-x}{y}$ $\frac{d^2y}{dx^2} = - \frac{[y \frac{dx}{dx} - x \frac{dy}{dx}]}{y^2}$ $y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$</p>	<p>1 1 1 1 2</p>
27	<p>$\frac{dy}{dx} = \frac{1}{2\sqrt{4+\sqrt{4+\sqrt{4+x^2}}}} \times \frac{1}{2\sqrt{4+\sqrt{4+x^2}}} \times \frac{2x}{2\sqrt{4+x^2}}$ $= \frac{x}{4\sqrt{4+\sqrt{4+\sqrt{4+x^2}}}\sqrt{4+\sqrt{4+x^2}}\sqrt{4+x^2}}$</p>	<p>1+1+2 2</p>
OR	<p>put $x = a \cos\theta \Rightarrow \frac{dx}{d\theta} = -a \sin\theta$ now $y = \frac{\sqrt{a(1+\cos\theta)} + \sqrt{a(1-\cos\theta)}}{\sqrt{a(1+\cos\theta)} - \sqrt{a(1-\cos\theta)}} = \frac{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}$ $= \tan(\frac{\pi}{4} + \frac{\theta}{2})$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\sec^2(\frac{\pi}{4} + \frac{\theta}{2}) \cdot (\frac{1}{2})}{-a \sin\theta} = - \frac{\sec^2(\frac{\pi}{4} - \frac{\theta}{2})}{2a \sin\theta}$</p>	<p>1 2 1 $\frac{1}{2} + 1 + \frac{1}{2}$</p>

28	<p>let r be radius of sphere and dimensions of cuboid are $\frac{x}{3}, x, 2x$ now $4\pi r^2 + 2\left(\frac{x}{3} \times x + x \times 2x + 2x \times \frac{x}{3}\right) = k$ (constant) $\Rightarrow r = \sqrt{\frac{k-6x^2}{4\pi}}$ Sum of volumes $V_S = \frac{4}{3}\pi r^3 + \frac{2x^3}{3} = \frac{4}{3}\pi \left(\frac{k-6x^2}{4\pi}\right)^{\frac{3}{2}} + \frac{2}{3}x^3$ $\frac{dV_S}{dx} = \frac{4}{3}\pi \times \frac{3}{2} \left(\frac{k-6x^2}{4\pi}\right)^{\frac{1}{2}} \left(-\frac{12x}{4\pi}\right) + \frac{2}{3} \times 3x^2 = -6x \sqrt{\frac{k-6x^2}{4\pi}} + 2x^2$ For max. or min $\frac{dV_S}{dx} = 0 \Rightarrow -6x + \sqrt{\frac{k-6x^2}{4\pi}} + 2x^2 = 0$ now $\frac{d^2V}{dx^2} = \frac{3}{\sqrt{\pi}} \frac{12x^2 - k}{\sqrt{k-6x^2}} + 4x$ at $x = \frac{3}{2\sqrt{\pi}} \sqrt{k-6x^2}$ $\frac{d^2V}{dx^2} = \frac{x(27-2\pi)}{\pi} + 4x > 0$ Hence when $x = 3r$ V is minimum now $V_S = \frac{4}{3}\pi \left(\frac{k-6x^2}{4\pi}\right)^{\frac{3}{2}} + \frac{2}{3}x^3$ when $x = 3r$ $V_S = \left(\frac{4\pi + 54}{3}\right)r^3$</p>	<p>1 $\frac{1}{2}$ 1 1 $\frac{1}{2}$ 1 1 1</p>
29	<p>$\frac{dy}{dx} = -\sin(x+y) \cdot \left(1 + \frac{dy}{dx}\right)$ $\Rightarrow \frac{dy}{dx} = \frac{-\sin(x+y)}{1 + \sin(x+y)}$ slope of line $x + 2y = 0$ is $-\frac{1}{2}$ $\therefore \frac{-\sin(x+y)}{1 + \sin(x+y)} = -\frac{1}{2} \Rightarrow x+y = \frac{\pi}{2}$ or $-\frac{3\pi}{2}$ $\therefore y = \cos(x+y) = \cos \frac{\pi}{2} = \cos\left(-\frac{3\pi}{2}\right) = 0 \Rightarrow y = 0$ So $x = \frac{\pi}{2}$ or $-\frac{3\pi}{2}$ \therefore tangent at $\left(-\frac{3\pi}{2}, 0\right)$; $(y-0) = -\frac{1}{2}(x - \frac{\pi}{2})$ $2x + 4y + 3\pi = 0$ and at $\left(\frac{\pi}{2}, 0\right)$; $(y-0) = -\frac{1}{2}(x - \frac{\pi}{2})$ $2x + 4y - \pi = 0$</p>	<p>1 $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 1</p>

OR

	$f(x) = \frac{4 \sin x}{2 + \cos x} - x \Rightarrow f'(x) = \frac{\cos x (4 - \cos x)}{(2 + \cos x)^2}$ <p>Clearly $\frac{4 - \cos x}{(2 + \cos x)^2} > 0 \forall x \in [0, 2\pi]$</p> <p>$\therefore$ Change of sign of $f'(x)$ depends on $\cos x$</p>	$\frac{1}{2}$ 1
	$f'(x) = 0 \Rightarrow \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \in [0, 2\pi]$  <p>Clearly $f(x)$ is increasing in $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$ and decreasing in $(\frac{\pi}{2}, \frac{3\pi}{2})$</p>	1 $\frac{1}{2}$ $\frac{1}{2}$